BEBR
FACULTY WORKING
PAPER NO. 90-1655

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May 1990

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* On different occasions I have benefited from discussions, comments and suggestions by Roko Aliprantis, Kim Border, Don Brown, Mark Feldman, Leo Hurwicz, Charlie Kahn, John Ledyard, Andreu Mas-Colell, Flavio Menezes, Tom Palfrey, Ed Prescott, Aldo Rustichini, David Schmeidler and Sanjay Srivastava. Mark Feldman and Aldo Rustichini both independently brought to my attention the related work of Wilson (1978). My thanks are extended to all the above individuals. Of course, I am responsible for any remaining shortcomings.

ABSTRACT

We introduce a new core concept for an exchange economy with differential information which is contained in the coarse core concept of Wilson (1978). We prove the existence of (i) a core allocation for an exchange economy with differential information and (ii) an α-core strategy for a game in normal form with differential information.

1. INTRODUCTION

An exchange economy with differential information, consists of a finite set of agents each of whom is characterized by a random utility function, an initial endowment, a private information set and a prior.

The purpose of this paper is to study the following questions: How does one define the notion of the core in an exchange economy with differential information? What is the appropriate core concept? Under what conditions on agent's characteristics is the core nonempty?

It should be noted that with finitely many states of nature, the existence of core allocations for an economy with differential information, follows easily from the well known result of Scarf (1967), as first shown in a seminal paper by Wilson (1978). However, with a continuum of states even if there is symmetric information (i.e., the information set of each agent is the same) the domain of the expected utility becomes infinite dimensional (even if there is only one good in the economy), and consequently Scarf's theorem is not directly applicable. It turns out that in the presence of a continuum of states, functional analytic methods as well as several measure theoretic results seem to be required.

The paper is organized as follows: Section 2 contains notation and definitions. The model and the main results are presented in Section 3. Sections 4 and 5 contain the proof of our main theorems. Finally Section 6 contains some concluding remarks.

2. NOTATION AND DEFINITIONS

2.1. Notation

$\mathbf{R}^l$ denotes the l-fold Cartesian product of the set of real numbers $\mathbf{R}$.

$\mathbf{R}^l_+$ denotes the positive cone of $\mathbf{R}^l$.

$\mathbf{R}^l_{++}$ denotes the strictly positive elements of $\mathbf{R}^l$.

2^A denotes the set of all nonempty subsets of the set A.

$\oslash$ denotes the empty set.

$/$ denotes the set theoretic subtraction.

If X is a linear topological space, its dual is the space X^* of all continuous linear functionals on X, and if $p \in X^*$, and $x \in X$ the value of p at x is denoted by $p \cdot x$.

2.2. Definitions

If X and Y are sets, the *graph* of the set-valued function (or correspondence), $\phi : X \to 2^Y$ is denoted by $G_\phi = \{(x, y) \in X \times Y : y \in \phi(x)\}$. Let $(T, \mathbf{T}, \mu)$ be a complete, finite measure space, and X be a separable Banach space. The set-valued function $\phi : T \to 2^X$ is said to have a *measurable graph* if $G_\phi \in \mathbf{T} \otimes \beta(X)$, where $\beta(X)$ denotes the Borel σ-algebra on X and $\otimes$ denotes product σ-algebra. The set-valued function $\phi : T \to 2^X$ is said to be *lower measurable* or just *measurable* if for every open subset V of X, the set $\{t \in T : \phi(t) \cap V \neq \oslash\}$ is an element of $\mathbf{T}$. A well-known result of Debreu [(1966), p. 359] says that if $\phi : T \to 2^X$ has a measurable graph, then ϕ is lower measurable. Furthermore, if $\phi(\cdot)$ is closed valued and lower measurable then $\phi : T \to 2^X$ has a measurable graph. A theorem of Aumann (1967) which will be of fundamental importance in this paper tells us, that if $(T, \mathbf{T}, \mu)$ is a complete, finite measure space, X is a separable metric space and $\phi : T \to 2^X$ is a nonempty valued correspondence having a measurable graph, then $\phi(\cdot)$ admits a *measurable selection*, i.e., there exists a measurable function $f : T \to X$ such that $f(t) \in \phi(t)$ μ*-a.e.*

Let $(T, \mathbf{T}, \mu)$ be a finite measure space and X be a Banach space. Following Diestel-Uhl (1977) the function $f : T \to X$ is called *simple* if there exist $x_1, x_2, \ldots, x_n$ in X and $\alpha_1, \alpha_2, \ldots, \alpha_n$ in $\mathbf{T}$ such that $f = \sum_{i=1}^{n} x_i \chi_{\alpha_i}$, where $\chi_{\alpha_i}(t) = 1$ if $t \in \alpha_i$ and $\chi_{\alpha_i}(t) = 0$ if $t \notin \alpha_i$. A function $f : T \to X$ is said to be μ*-measurable* if there exists a sequence of simple

functions $f_n : T \to X$ such that $\lim_{n\to\infty} \|f_n(t) - f(t)\| = 0$ for almost all $t \in T$. A μ*-measurable* function $f : T \to X$ is said to be *Bochner integrable* if there exists a sequence of simple functions $\{f_n : n = 1, 2, \ldots\}$ such that

$$\lim_{n\to\infty} \int_T \|f_n(t) - f(t)\| d\mu(t) = 0.$$

In this case we define for each $E \in \mathrm{T}$ the integral to be $\int_E f(t) d\mu(t) = \lim_{n\to\infty} \int_E f_n(t) d\mu(t)$. It can be shown [see Diestel-Uhl (1977), Theorem 2, p. 45] that, if $f : T \to X$ is a μ-measurable function then f is Bochner integrable if and only if $\int_T \|f(t)\| d\mu(t) < \infty$. It is important to note that the *Dominated Convergence Theorem* holds for Bochner integrable functions, in particular, if $f_n : T \to X$, $(n = 1, 2, \ldots)$ is a sequence of Bochner integrable functions such that $\lim_{n\to\infty} f_n(t) = f(t)$ μ*-a.e.*, and $\|f_n(t)\| \leq g(t)$ μ*-a.e.*, (where $g : T \to \mathbf{R}$ is an integrable function), then f is Bochner integrable and $\lim_{n\to\infty} \int_T \|f_n(t) - f(t)\| d\mu(t) = 0$.

For $1 \leq p < \infty$, we denote by $L_p(\mu, X)$ the space of equivalence classes of X-valued Bochner integrable functions $x : T \to X$ normed by

$$\|x\|_p = \left(\int_T \|x(t)\|^p d\mu(t)\right)^{\frac{1}{p}}.$$

It is a standard result that normed by the functional $\|\cdot\|_p$ above, $L_p(\mu, X)$ becomes a Banach space [see Diestel-Uhl (1977), p. 50]. Recall that a correspondence $\phi : T \to 2^X$ is said to be *integrably bounded* if there exists a map $h \in L_1(\mu, R)$ such that

$\sup\{\|x\| : x \in \phi(t)\} \leq h(t)$ μ*-a.e.*

A Banach space X has the *Radon–Nikodym Property with respect to the measure space* (T, T, μ) if for each μ-continuous measure $G : \mathrm{T} \to \mathrm{X}$ of bounded variation there exists $g \in L_1(\mu, X)$ such that $G(E) = \int_E g(t) d\mu(t)$ for all $E \in \mathrm{T}$. A Banach space X has the *Radon-Nikodym property* (RNP) if X has the RNP with respect to every finite measure space. Recall now [see Diestel-Uhl (1977, Theorem 1, p. 98)] that if (T, T, μ) is a finite measure space $1 \leq p < \infty$, and X is a Banach space, then X^* has the RNP if and only if

$(L_p(\mu, X))^* = L_q(\mu, X^*)$ where $\frac{1}{p} + \frac{1}{q} = 1$.

We will close this section by collecting some basic results on Banach lattices [for an excelent treatment see Aliprantis-Burkinshaw (1985)]. Recall that a *Banach lattice* is a Banach space L equipped with an order relation $\geq$ (*i.e.*, $\geq$ is a reflexive, antisymmetric and transitive relation) satisfying:

(i) $x \geq y$ implies $x + z \geq y + z$ for every z in L,

(ii) $x \geq y$ implies $\lambda x \geq \lambda y$ for all $\lambda \geq 0$,

(iii) for all x, y in L there exists a supremum (least upper bound) $x \vee y$ and an infimum (greatest lower bound) $x \wedge y$,

(iv) $|x| \geq |y|$ implies $\|x\| \geq \|y\|$ for all x, y in L.

As usual $x^+ = x \vee 0$, $x^- = (-x) \vee 0$ and $|x| = x \vee (-x) = x^+ + x^-$; we call x^+, x^- the *positive* and *negative parts* of x, respectively and $|x|$ the *absolute value* of x. The symbol $\|\cdot\|$ denotes the norm on L. If x, y are elements of the Banach lattice L, then we define the *order interval* $[x, y]$ as follows:

$$[x, y] = \{z \in L : x \leq z \leq y\}.$$

Note that $[x, y]$ is norm closed and convex (hence weakly closed). A Banach lattice L is said to have an *order continuous norm* if, $x_\alpha \downarrow 0$ in L implies $\|x_\alpha\| \downarrow 0$. A very useful result which is going to play an important role in the sequel is that if L is a Banach lattice then the fact that L has order continuous norm is equivalent to the weak compactness of the order interval $[x, z] = \{y \in L : x \leq y \leq z\}$ for every x, z in L, [see for instance Aliprantis-Brown-Burkinshaw (1989), Theorem 2.3.8, p. 104 or Lindenstrauss-Tzafriri (1979, p. 28)].

We finally note that Cartwright (1974) has shown that if X is a Banach lattice with order continuous norm (or equivalenty X has weakly compact order intervals) then $L_1(\mu, X)$, has weakly compact order intervals, as well. *Cartwright's theorem* will play a crucial role in the proof of our main results.

3. MODEL AND RESULTS

3.1 The Core of an Exchange Economy with Differential Information

Let Y be a separable Banach lattice with order continuous norm, whose dual Y^* has the RNP.[1] Let $(\Omega, \mathbf{F}, \mu)$ be a complete σ-finite measure space.

An *exchange economy with differential information* $\Gamma = \{(X_i, u_i, e_i, F_i, q_i) : i = 1,2,\ldots, n\}$ is a set of quintuples $(X_i, u_i, e_i, F_i, q_i)$ where,

(1) $X_i : \Omega \to 2^{Y_+}$ is the *random consumption set* of agent i,

(2) $u_i : \Omega \times X_i \to \mathbf{R}$ is the *random utility function* of agent i,

(3) F_i is a (measurable) partition[2] of $(\Omega, \mathbf{F})$ denoting the *private information* of agent i,

(4) $e_i : \Omega \to Y_+$ is the *random initial endowment* of agent i, $e_i(\cdot)$ is F_i-measurable, Bochner integrable and $e_i(\omega) \in X_i(\omega)$ for all i, μ-*a.e.*,

(5) $q_i : \Omega \to \mathbf{R}_{++}$ is the *prior* of agent i, (i.e., q_i is a Radon-Nikodym derivative having the property that $\int_{t\in\Omega} q_i(t)d\mu(t) = 1$).

Denote by L_{X_i} the *set of all Bochner integrable and F_i-measurable selections from the consumption set* X_i of agent i, i.e.,

$$L_{X_i} = \{x_i \in L_1(\mu, Y_+) : x_i : \Omega \to Y_+ \text{ is } F_i\text{-measurable and } x_i(\omega) \in X_i(\omega) \ \mu\text{-}a.e.\}.$$

For each i, $(i = 1, 2, \ldots, n)$, denote by $E_i(\omega)$ the event in F_i containing the realized state of nature $\omega \in \Omega$ and suppose that $\int_{t\in E_i(\omega)} q_i(t)d\mu(t) > 0$ for all i. Given $E_i(\omega)$ in F_i define the *conditional expected utility* of agent i, $V_i : \Omega \times L_{X_i} \to \mathbf{R}$ by

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[1] A basic example of a space which satisfies all these conditions is the Euclidean space $\mathbf{R}^l$. Remark 6.1 in Section 6 presents some more examples.

[2] One could have assumed that F_i is a sub σ-algebra of $\mathbf{F}$.

$$(3.1) \qquad V_i(\omega, x_i) = \int_{t \in E_i(\omega)} u_i(t, x_i(t)) q_i(t \mid E_i(\omega)) d\mu(t)$$

where

$$(3.2) \qquad q_i(t \mid E_i(\omega)) = \begin{cases} 0 & \text{if } t \notin E_i(\omega) \\ \dfrac{q_i(t)}{\int_{t \in E_i(\omega)} q_i(t) d\mu(t)} & \text{if } t \in E_i(\omega) . \end{cases}$$

We are now ready to define the central notions of the paper.

Definition 3.1.1: We say that $x = (x_1, x_2, \ldots, x_n) \in \prod_{i=1}^{n} L_{X_i}$ is a *core allocation* for Γ, if

(i) $\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} e_i$, and

(ii) It is not true that there exist $S \subset \{1, 2, \ldots, n\}$ and $(y_i)_{i \in S} \in \prod_{i \in S} L_{X_i}$ such that $\sum_{i \in S} y_i = \sum_{i \in S} e_i$ and $V_i(\omega, y_i) > V_i(\omega, x_i)$ for all $i \in S$ for μ-almost all $\omega \in \Omega$ (where V_i is given by (3.1)).

A couple of comments are in order: Note that $x \in \prod_{i=1}^{n} L_{X_i}$ implies that each $x_i(\cdot)$ is F_i-measurable and therefore the vector $x(\omega) = (x_1(\omega), x_2(\omega), \ldots, x_n(\omega)) \in \prod_{i=1}^{n} X_i(\omega)$ is $\bigvee_{i=1}^{n} F_i$-measurable, (where $\bigvee_{i=1}^{n} F_i$ denotes the join, the smallest partition containing $F_1, F_2, \ldots, F_n$). Condition (i) above implies that the markets are cleared in each state of nature, i.e., $\sum_{i=1}^{n} x_i(\omega) = \sum_{i=1}^{n} e_i(\omega)$ $\mu - a.e.$. Condition (ii) shows that no coalition of agents (while each agent in the coalition uses his/her own private information) can redistribute their initial endowments among themselves for any state of nature and make the conditional expected utility of each agent in the coalition better off.

Note that condition (ii) of definition 3.1.1. implies the following condition:

(ii)′ It is not true that there exist $S \subset \{1,2, ..., n\}$ and $y : \Omega \to \prod_{i \in S} X_i$, $y_i(\cdot)$ is $\bigwedge_{i \in S} F_i$–measurable (where $\bigwedge_{i \in S} F_i$ denotes the meet, i.e., the maximal partition contained in all of them) such that $\sum_{i \in S} y_i(\omega) = \sum_{i \in S} e_i(\omega)$ μ–$a.e.$ and $V_i(\omega, y_i) > V_i(\omega, x_i)$ for all $i \in S$ for μ–almost all $\omega \in \Omega$.

The above blocking notion is the one adopted by Wilson (1978) to define his *coarse core* concept.[3] Note that since each $y_i(\cdot)$ is $\bigwedge_{i \in S} F_i$–measurable, the information is verifiable by each member of the coalition. For instance, if we imagine that agents negotiate the terms of a contract, then Wilson's definition tells us that a coarse core allocation has the property that no coalition of agents can exchange their own information (in fact, information is verifiable by each member of the coalition) and make each agent in the coalition better off. In other words contracts are realizable because information is verifiable. However, according to our condition (ii) of definition 3.1.1., information is not necessarily verifiable by all the members of the coalition (it is only privately verifiable). The latter makes the core smaller, i.e., any core allocation satisfying the definition 3.1.1. is a coarse core allocation (recall that if $y_i(\cdot)$ is $\bigwedge_{i \in S} F_i$–measurable, it is also F_i–measurable, of course the reverse is not true). Hence, the theorems that we will prove on the existence of core allocations will imply the existence of coarse core allocations as well.

Note that if we were to narrow the set of core allocations by replacing F_i-measurability of $y_i(\cdot)$ in (ii) of definition 3.1.1, with the $\bigvee_{i \in S} F_i$-measurability of $y : \Omega \to \prod_{i \in S} X_i$, then it is easy to construct examples which satisfy all the assumptions of Theorem 3.1 below, but the core is empty [see Wilson (1978) for examples to that effect]. We are not aware of any natural set of assumptions on utility functions and initial endowments which will guarantee the existence of such a core. Finally, it is worth pointing out that a core notion which allows for complete

[3] See also Kobayashi (1980) who has also used the coarse core.

exchange of information among agents in each coalition may not be an appropriate concept, since in most applications, agents do not have an incentive to reveal their own private information (think of situations of moral hazard or adverse selection).

Definition 3.1.2: We say that $x \in \prod_{i=1}^{n} L_{X_i}$ is (interim) *Pareto optimal if:*[4]

(i) $\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} e_i$, and

(ii) It is not true that there exists $y \in \prod_{i=1}^{n} L_{X_i}$ such that $\sum_{i=1}^{n} y_i = \sum_{i=1}^{n} e_i$ and $V_i(\omega, y_i) > V_i(\omega, x_i)$ for all i for μ–almost all $\omega \in \Omega$ (where V_i is given by (3.1)).

Definition 3.1.3: We say that $x \in \prod_{i=1}^{n} L_{X_i}$ is *individually rational* if:

(i) $\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} e_i$ and

(ii) $V_i(\omega, x_i) \geq V_i(\omega, e_i)$ for all i and for μ–almost all $\omega \in \Omega$, (where V_i is given by (3.1)).

Finally, if the private information set of each agent, is the same (symmetric information, i.e., $F_i \equiv F$ for all i) we call any $x \in \prod_{i=1}^{n} L_{X_i}$ satisfying (i) and (ii) of Definition 3.1.1, *symmetric core allocation* for Γ.

We are now ready to state our first main result:

Theorem 3.1: Let $\Gamma = \{(X_i, u_i, e_i, F_i, q_i) : i = 1, 2, \ldots, n\}$ be an exchange economy with differential information satisfying the following assumptions, for each

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[4] A similar notion is defined by Palfrey and Srivastava (1987).

i, $(i = 1, 2, \ldots, n)$,

(a.3.1) $X_i : \Omega \to 2^{Y_+}$ is an integrably bounded, convex, closed, nonempty valued and F_i-measurable correspondence,

(a.3.2) for each $\omega \in \Omega$, $u_i(\omega, \cdot)$ is weakly continuous and integrably bounded, and

(a.3.3) for each $\omega \in \Omega$, $u_i(\omega, \cdot)$ is quasi-concave.

Then a core allocation exists in Γ.

The following Corollaries follow directly from Theorem 3.1.

Corollary 3.1: Let $\Gamma = \{(X_i, u_i, e_i, F_i, q_i) : i = 1, 2, \ldots, n\}$ be an exchange economy with differential information satisfying all the assumptions of Theorem 3.1. Then an individually rational and Pareto optimal allocation exists in Γ.

Corollary 3.2: Let $\Gamma = \{(X_i, u_i, e_i, F_i, q_i) : i = 1, 2, \ldots, n\}$ be an exchange economy with symmetric information, (i.e., $F_i \equiv F$ for all i) satisfying all the assumptions of Theorem 3.1. Then a symmetric core allocation exists in Γ.

3.2. The α−Core of a Game in Normal Form with Differential Information

A *game in normal form with differential information* $B = \{(X_i, u_i, F_i, q_i) : i = 1, 2, \ldots, n\}$ is a set of quadruples (X_i, u_i, F_i, q_i) where

(1) $X_i : \Omega \to 2^Y$ is the *strategy set-valued function* of player i,

(2) $u_i : \Omega \times \prod_{i=1}^{n} X_i \to \mathbf{R}$ is the *random payoff function* of player i,

(3) F_i is a (measurable) partition of $(\Omega, \mathbf{F})$ denoting the *private information* of player i, and

(4) $q_i : \Omega \to \mathbf{R}_{++}$ is the *prior* of player i, (i.e., q_i is a Radon-Nikodym derivative having the property that $\int_{t\in\Omega} q_i(t)d\mu(t) = 1$).

For each i, ($i = 1, 2, \ldots, n$) denote by $E_i(\omega)$ the event in F_i containing the true state of nature $\omega \in \Omega$ and suppose that $\int_{t\in E_i(\omega)} q_i(t)d\mu(t) > 0$. Given $E_i(\omega)$ in F_i define the *conditional expected payoff* of player i, $V_i : \Omega \times \prod_{i=1}^{n} L_{X_i} \to \mathbf{R}$ *by*

$$(3.3) \qquad V_i(\omega, x) = \int_{t\in E_i(\omega)} u_i(t, x(t))q_i(t \mid E_i(\omega))d\mu(t),$$

where $q_i(t \mid E_i(\omega))$ is defined as in (3.2).

Before we define the notion of an *α-core strategy for the game* **B** we need to introduce some notation. Denote by I the set of players $\{1, 2, \ldots, n\}$. If $S \subset I$ then $(y^S, x^{I/S})$ denotes the vector z in $\prod_{i=1}^{n} L_{X_i}$ where $z_i = y_i$ if $i \in S$ and $z_i = x_i$ if $i \notin S$.

Definition 3.2.1: We say that $x \in \prod_{i=1}^{n} L_{X_i} = L_X$ is an *α-core strategy* for B if:

(i) It is not true that there exist $S \subset I$ and $(y_i)_{i\in S} \in \prod_{i\in S} L_{X_i}$ such that for any $z^{I/S} \in \prod_{i\notin S} L_{X_i}$, $V_i(\omega, (y^S, z^{I/S})) > V_i(\omega, x)$ for all $i \in S$ for μ-almost all $\omega \in \Omega$ (where V_i is given by (3.3)).

Note that as previously $x \in \prod_{i=1}^{n} L_{X_i}$ implies that each $x_i(\cdot)$ is F_i-measurable and consequently the vector $x(\omega) = (x_1(\omega), \ldots, x_n(\omega))$ is $\bigvee_{i\in I} F_i$-measurable. Condition (i) in Definition 3.2.1 indicates that no coalition of players is able to change its strategy (while each player in the coalition uses his/her own private information) and make the expected utility of each member in the coalition better off, no matter what the complementary coalition chooses to do

(each member in the complementary coalition is also allowed to take advantage of his/her own private information). Following the previous definition of a coarse core allocation for an economy with differential information, we can define an α-coarse strategy for the game B, and show that the set of α-coarse core strategies contains the set of α-core strategies for the game B.

Since there is no exchange of information among players in each coalition one may suggest that it is possible to analyze games in normal form with differential information (or economies with differential information) in a noncooperative setting adopting the notion of a Bayesian Nash equilibrium or correlated equilibrium. However, the latter concepts do not yield Pareto optimal outcomes, contrary to the core or α-core. It seems to us that selecting outcomes out of the Pareto frontier is an attractive property for an allocation mechanism to have. The latter makes the core concept appealing in an economy with differential information.

We can now state our second main result.

Theorem 3.2: Let $B = \{(X_i, u_i, F_i, q_i) : i = 1,2,\ldots, n\}$ be a game in normal form with differential information satisfying the following assumptions for each player i, $(i = 1, 2, \ldots, n)$,

(a.3.2.1) $X_i : \Omega \to 2^{Y_+}$ is an integrably bounded, nonempty convex weakly compact valued and F_i-measurable correspondence,[5]

(a.3.2.2) for each $\omega \in \Omega$, $u_i(\omega, \cdot)$ is weakly continuous and integrably bounded, and

(a.3.2.3) for each $\omega \in \Omega$, $u_i(\omega, \cdot)$ is quasi concave.

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[5] The assumption that $X_i(\cdot)$ takes values in the positive cone of Y, is not needed for the proof of this theorem.

Then an α-core strategy exists in B.

4. PROOF OF THEOREM 3.1

We first state the well-known core existence result of Scarf (1967) [see also Border (1984) or Yannelis (1985) for recent generalizations] which is going to play a crucial role in the proof of Theorem 3.1. We will first need some notation.

Let $\mathbf{E} = \{(X_i, u_i, e_i) : i = 1, 2, \ldots, n\}$ be an exchange economy, where

(1) $X_i \subset \mathbf{R}^l$ is the *consumption set* of agent i,

(2) $u_i : X_i \to \mathbf{R}$ is the *utility function* of agent i, and

(3) $e_i \in X_i$ is the *initial endowment* of agent i

Define the set-valued function $P_i : X_i \to 2^{X_i}$ by $P_i(x_i) = \{y_i \in X_i : u_i(y_i) > u_i(x_i)\}$. *Scarf's result* asserts that if X_i is a nonempty, closed, convex and bounded from below subset of $\mathbf{R}^l$, u_i is quasi concave and continuous, (i.e., if P_i is convex valued and has an open graph in $X_i \times X_i$), then core allocations exist in $\mathbf{E}$, i.e., there exists $x \in \prod_{i=1}^{n} X_i$ such that:

(i) $\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} e_i$, and

(ii) it is not true that there exist $S \subset \{1, 2, \ldots, n\}$ and $(y_i)_{i\in S} \in \prod_{i\in S} X_i$ such that $\sum_{i\in S} y_i = \sum_{i\in S} e_i$ and $y_i \in P_i(x_i)$ for all $i \in S$.

We begin the proof of Theorem 3.1 by constructing a new economy $G = \{(L_{X_i}, P_i, e_i) : i = 1, 2, \ldots, n\}$ where,

(i) L_{X_i} is the *consumption set* of agent i,

(ii) $P_i : L_{X_i} \to 2^{L_{X_i}}$ is the *preference correspondence* of agent i defined by

$P_i(x_i) = \{y_i \in L_{X_i} : V_i(\omega, y_i) > V_i(\omega, x_i)$ for μ-almost all $\omega \in \Omega\}$, and

(iii) $e_i \in L_{X_i}$ for all i, is the *initial endowment* of agent i.

Note that the existence of a core allocation for G implies the existence of a core allocation for the original economy $\Gamma = \{(X_i, u_i, e_i, F_i, q_i) : i = 1, 2, \ldots, n\}$. Hence, all we need to show is that a core allocation exists in the economy G. To this end we first show that for each i, L_{X_i} is closed, bounded convex, nonempty and that $P_i : L_{X_i} \to 2^{L_{X_i}}$ is convex valued having a weakly open graph (i.e., the set $G_{P_i} = \{(x, y) \in L_{X_i} \times L_{X_i} : y \in P_i(x)\}$ is weakly open in $L_{X_i} \times L_{X_i}$).

Note that the fact that L_{X_i} is convex, closed and bounded follows directly from assumption (a.3.1). To prove that L_{X_i} is nonempty, recall that $X_i : \Omega \to 2^{Y_+}$ is F_i-measurable, nonempty, closed valued and therefore $G_{X_i} \in F_i \otimes \beta(Y_+)$. By the Aumann (1967) measurable selection theorem, we can obtain an F_i-measurable function $f_i : \Omega \to Y_+$ such that $f_i(\omega) \in X_i(\omega)$ μ*–a.e.* Since X_i is integrably bounded, we can conclude that $f_i \in L_1(\mu, Y_+)$. Hence, $f_i \in L_{X_i}$ and this proves that L_{X_i} is nonempty.

In order to show that for each i, P_i has a weakly open graph, we will first need the following claim:[6]

Claim 4.1 For each i, (i = 1, 2, ...) and for each $\omega \in \Omega$, $V_i(\omega, \cdot)$ is weakly continuous.

•

Proof: Fix i, ($i = 1, 2, \ldots, n$) and $\omega \in \Omega$ and let $E_i(\omega)$ be an event in F_i. Consider the sequence $\{x_i^m : m = 1, 2, \ldots\}$ in $L_{X_i} \subset L_1(\mu, Y)$, which converges weakly to $x_i \in L_{X_i}$, i.e., $p \cdot x_i^m$ converges to $p \cdot x_i$ for any $p \in L_\infty(\mu, Y^*) = (L_1(\mu, Y))^*$ (recall that Y^* has the RNP). Note that x_i^m converges weakly to x_i is equivalent to the fact that $p \cdot x_i^m \chi_A = p\chi_A \cdot x_i^m$ converges to $p \cdot x_i \chi_A = p\chi_A \cdot x_i$ for any $p \in L_\infty(\mu, Y^*)$, $A \in \mathbf{F}$ and each condition above implies that $y^* \cdot x_i^m \chi_A = y^* \chi_A \cdot x_i^m$ converges to $y^* \cdot x_i \chi_A = y^* \chi_A \cdot x_i$ for any $y^* \in Y^*$, $A \in \mathbf{F}$. If we

[6] A similar result is proved in Yannelis-Rustichini (1988).

show that $x_i^m \chi_{E_i(\omega)}$ converges pointwise in the weak topology of X_i to $x_i \chi_{E_i(\omega)}$, then since for each $\omega \in \Omega$, $u_i(\omega, \cdot)$ is weakly continuous and integrably bounded the weak continuity of $V_i(\omega, \cdot)$ will follow from the Lebesgue dominated convergence theorem. Now if $F_i = \{E_i^1, E_i^2, \ldots\}$ is the partition of agent i, then the fact that x_i^m and x_i are elements of L_{X_i} implies that $x_i^m = \sum_{k=1}^{\infty} x_{i,k}^m \chi_{E_i^k}$, $x_i = \sum_{k=1}^{\infty} x_{i,k} \chi_{E_i^k}$, for $x_{i,k}^m$, $x_{i,k}$ in X_i and consequently we can conclude that $x_i^m \chi_{E_i(\omega)} = \sum_{k=1}^{\infty} x_{i,k}^m \chi_{E_i^k \cap E_i(\omega)}$ converges weakly to $x_i \chi_{E_i(\omega)} = \sum_{k=1}^{\infty} x_{i,k} \chi_{E_i^k \cap E_i(\omega)}$. This completes the proof of the claim.

In view of Claim 4.1 we can now conclude that for each i, P_i has a weakly open graph. Moreover, since for each $\omega \in \Omega$, $u_i(\omega, \cdot)$ is quasi-concave so is $V_i(\omega, \cdot)$ and therefore, P_i is convex valued. We will now construct a suitable family of truncated subeconomies in a finite dimensional commodity space, each of which satisfies the assumptions of Scarf's theorem. Applying Scarf's theorem, we will obtain a net of core allocations for each subeconomy. By taking limits we will show that the existence of a core allocation for each subeconomy implies the existence of a core allocation for the original economy G.

Let A be the set of all finite dimensional subspaces of $L_1(\mu, Y_+)$ containing the initial endowments. For each $\alpha \in$ A define the consumption set of agent i, $L_{X_i}^{\alpha}$ by $L_{X_i}^{\alpha} = L_{X_i} \cap \alpha$ and the preference correspondence of agent i, $P_i^{\alpha}: L_{X_i}^{\alpha} \to 2^{L_{X_i}^{\alpha}}$ by $P_i^{\alpha}(x_i) = P_i(x_i) \cap L_{X_i}^{\alpha}$. We now have an economy $G^{\alpha} = \{(L_{X_i}^{\alpha}, P_i^{\alpha}, e_i) : i = 1, 2, \ldots, n\}$ in finite dimensional commodity space, where,

(4.1) $L_{X_i}^{\alpha}$ is the consumption set of agent i,

(4.2) $P_i^{\alpha}: L_{X_i}^{\alpha} \to 2^{L_{X_i}^{\alpha}}$ is the preference correspondence of agent i, and

(4.3) $e_i \in L_{X_i}^{\alpha}$ is the initial endowment of agent i.

It can be easily checked that each economy G^{α} satisfies all the ssumptions of Scarf's theorem and therefore there exists $x^{\alpha} = (x_1^{\alpha}, \ldots, x_n^{\alpha}) \in \prod_{i=1}^{n} L_{X_i}^{\alpha} = L_X^{\alpha}$ such that :

(4.4) $\sum_{i=1}^{n} x_i^{\alpha} = \sum_{i=1}^{n} e_i$, and

(4.5) it is not true that there exist $S \subset \{1, 2, \ldots, n\}$ and $(y_i)_{i \in S} \in \prod_{i \in S} L_{X_i}^{\alpha}$ such that $\sum_{i \in S} y_i = \sum_{i \in S} e_i$ and $y_i \in P_i^{\alpha}(x_i^{\alpha})$ for all $i \in S$.

From (4.4) it follows that for each $\alpha \in A$

$$0 \leq \sum_{i=1}^{n} x_i^{\alpha} = \sum_{i=1}^{n} e_i = e.$$

Hence for each $\alpha \in A$ the vectors x_i^{α} lie in the order interval $[0, e] = \{z \in \prod_{i=1}^{n} L_{X_i} : 0 \leq z \leq e\}$. Since by assumption order intervals in Y are weakly compact, by Cartwright's theorem the order interval $[0, e]$ in $\prod_{i=1}^{n} L_{X_i}$ is weakly compact. Direct the set A by inclusion so that $\{(x_1^{\alpha}, x_2^{\alpha}, \ldots, x_n^{\alpha}) : \alpha \in A\}$ forms a net in $\prod_{i=1}^{n} L_{X_i}$. Since all the vectors x_i^{α} lie in the order interval $[0, e]$ which is weakly compact, the net $\{(x_1^{\alpha}, \ldots, x_n^{\alpha}) : \alpha \in A\}$ has a subnet which converges weakly to some vector $x_1, x_2, \ldots, x_n$ in $[0, e]$. We will show that the vector $x_1, \ldots, x_n$ is a core allocation for the economy G. Denote the convergent subnet by $\{(x_1^{\alpha(m)}, \ldots, x_n^{\alpha(m)}) : m \in M\}$ where M is a set directed by "$\geq$". Since for all $m \in M$, $\sum_{i=1}^{n} x_i^{\alpha(m)} = \sum_{i=1}^{n} e_i$ and $x_i^{\alpha(m)}$ converges weakly to $x_i \in L_{X_i}$, we conclude that $\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} e_i$ We will now complete the proof by showing that:

(4.6) It is not true that there exist $S \subset \{1, 2, \ldots, n\}$ and $(y_i)_{i \in S} \in \prod_{i \in S} L_{X_i}$ such that $\sum_{i \in S} y_i = \sum_{i \in S} e_i$ and $y_i \in P_i(x_i)$ for all $i \in S$.

Suppose that (4.6) is false, then there exist $S \subset \{1, 2, \ldots, n\}$ and $(y_i)_{i \in S} \in \prod_{i \in S} L_{X_i}$ such that $\sum_{i \in S} y_i = \sum_{i \in S} e_i$ and $y_i \in P_i(x)$ for all $i \in S$. Since $x_i^{\alpha(m)}$ converges weakly to x_i and P_i has a weakly open graph, there exists $m_0 \in M$ such that $y_i \in P_i(x_i^{\alpha(m)})$ for all $m \geq m_0$ and for all $i \in S$. Choose $m_1 \geq m_0$ so that , if $m \geq m_1$, $y_i \in L_{X_i}^{\alpha(m)}$ for all $i \in S$. Then $y_i \in P_i^{\alpha(m)}(x_i^{\alpha(m)})$, for all $m \geq m_1$ and for all $i \in S$. But this contradicts (4.5). Hence (4.6) holds and this completes the proof of the theorem.

5. PROOF OF THEOREM 3.2

We begin by stating the α-core existence result of Scarf (1971) which is going to be used in the proof of Theorem 3.2.

Let $N = \{(X_i, u_i) : i = 1, 2, \ldots, n\}$ be a *game in normal form* where,

(1) X_i is a compact, convex and nonempty subset of $\mathbf{R}^l$, denoting the *strategy set* of player i, and

(2) $u_i : \prod_{i=1}^{n} X_i \to \mathbf{R}$ is a quasi-concave function on $\prod_{i=1}^{n} X_i$ denoting the *payoff* of player i.

The strategy vector $x \in \prod_{i=1}^{n} X_i$ is said to be an *α-core strategy* for N if:

It is not true that there exist $S \subset \{1, 2, \ldots, n\}$ and $(y_i)_{i \in S} \in \prod_{i \in S} X_i$ such that for any $z^{I/S} \in \prod_{i \notin S} X_i$, $u_i(y^S, z^{I/S}) > u_i(x)$ for all $i \in S$.

As in the proof of Theorem 3.1 we will construct a new game $\overline{\mathbf{B}} = \{(L_{X_i}, V_i) : i = 1, 2, \ldots, n\}$ where,

(a) L_{X_i} is the *strategy set* of player i, and

(b) $V_i : \Omega \times \prod_{i=1}^{n} L_{X_i} \to \mathbf{R}$ is the *payoff function* of player i, (defined as in (3.3)) .

It is easy to see that the existence of an α-core strategy for $\overline{\mathbf{B}}$ implies the existence of an α-core strategy for the original game $\mathbf{B} = \{(X_i, u_i, F_i, q_i) : i = 1, 2, \ldots, n\}$. Our goal is to construct a suitable family of truncated subgames in a finite dimensional strategy space, each of which satisfies all the conditions of the Scarf (1971) theorem. Therefore we will obtain a net of α-core strategies for each subgame. As in the proof of Theorem 3.1, operating a limiting argument we can show that the existence of an α-core strategy for each subgame implies the existence of an α-core strategy for the orginal game $\mathbf{B}$. Before we start the outlined construction of the family of truncated subgames, we need to make some observations.

Note that for each $\omega \in \Omega$, $V_i(\omega, \cdot\,)$ is weakly continuous (recall Claim 4.1) and by virtue of assumption (a.3.2.3) quasi-concave on $\prod_{i=1}^{n} L_{X_i}$. Moreover, note that each L_{X_i} is convex and nonempty. However, since Scarf's theorem requires the compactness of each strategy set we will need to prove the following claim which is known as Diestel's theorem.

Claim 5.1: The set L_{X_i} is weakly compact in $L_1(\mu, Y)$.

Proof: The proof is based on the celebrated theorem of James (1964) and it is patterned after that of Khan (1982). Note that the dual of $L_1(\mu, Y)$ is $L_\infty(\mu, Y^*_{w^*})$ (where w^* denotes the w^*-topology), i.e., $(L_1(\mu, Y))^* = L_\infty(\mu, Y^*_{w^*})$, [see, for instance, Tulcea-Tulcea (1969)]. Let x be an arbitrary element of $L_\infty(\mu, Y^*_{w^*})$. If we show that x attains its supremum on L_{X_i} the result will follow from James' theorem, [James (1964)]. Let,

$$\sup_{\psi_i \in L_{X_i}} \psi \cdot x = \sup_{\psi_i \in L_{X_i}} \int_{\omega\in\Omega} (\psi_i(\omega) \cdot x(\omega)) d\mu(\omega).$$

Note that by theorem 2.2 in Hiai-Umegaki (1977),

$$\sup_{\psi_i \in L_{X_i}} \int_{\omega\in\Omega} (\psi_i(\omega) \cdot x(\omega)) d\mu(\omega) = \int_{\omega\in\Omega} \sup_{\phi_i \in X_i(\omega)} (\phi_i \cdot x(\omega)) d\mu(\omega).$$

For each i, define the set-valued function $g_i : \Omega \to 2^Y$ by

$g_i(\omega) = \{y \in X_i(\omega) : y \cdot x = \operatorname{Sup}_{\phi_i \in X_i(\omega)} \phi_i \cdot x\}$. It follows from the weak compactness of X_i that for all $\omega \in \Omega$, $g_i(\omega)$ is nonempty. For each i, define $f_i : \Omega \times Y \to [-\infty, \infty]$ by $f_i(\omega, y) = y \cdot x - \operatorname{Sup}_{\phi_i \in X_i(\omega)} \phi \cdot x$. It is easy to see that for each fixed $\omega \in \Omega$, $f_i(\omega, \cdot\,)$ is continuous and for each fixed $y \in Y$, $f_i(\,\cdot\,, y)$ is F_i-measurable and hence $f(\,\cdot\,,\cdot\,)$ is jointly F_i-measurable, i.e., for every closed subset V of $[-\infty, \infty]$, $f_i^{-1}(V) = \{(\omega, z) \in \Omega \times Y : z \in X_i(\omega)\}$ belongs to $F_i \otimes \mathbf{B}(Y)$. Since X_i is F_i-measurable the set $G_{X_i} = \{(\omega, x) : x \in X_i(\omega)\}$ is an element of $F_i \otimes \mathbf{B}(Y)$. Moreover, note that $G_{g_i} = f_i^{-1}(0) \cap G_{X_i}$ and since $f_i^{-1}(0)$ and G_{X_i} belong to $F_i \otimes \mathbf{B}(Y)$ so does G_{g_i}. It follows from the Aumann measurable selection theorem that there exists an F_i-measurable function $z_i : \Omega \to Y$ such that $z_i(\omega) \in g_i(\omega)$ μ-*a.e.* Thus, $z_i \in L_{X_i}$ and $\operatorname{Sup}_{\phi_i \in L_{X_i}} \phi_i \cdot x = \int_{\omega \in \Omega} (z_i(\omega) \cdot x(\omega)) d\mu(\omega) = z_i \cdot x$. Since $x \in L_\infty(\mu, Y^*_{w^*})$ was arbitrarily chosen, we conclude that every element of $(L_1(\mu, Y))^*$ attains its supremum on L_{X_i}, and this completes the proof of the fact that L_{X_i} is weakly compact.

We are now ready to construct a suitable family of truncated subgames. To this end let Λ be a family of all finite subsets of L_{X_i}. For each $\lambda \in \Lambda$ let $L_{X_i}^{\lambda}$ denote the closed convex hull of λ. Then each $L_{X_i}^{\lambda}$ is a compact, convex, nonempty subset of a finite dimensional Euclidean space and $\bigcup_{\lambda \in \Lambda} L_{X_i}^{\lambda} = L_{X_i}$. Moreover, the set $\{L_{X_i}^{\lambda} : \lambda \in \Lambda\}$ is directed upwards by inclusion. For each $\lambda \in \Lambda$ we have a game $\overline{\mathbf{B}}^{\lambda} = \{(L_{X_i}^{\lambda}, V_i^{\lambda}) : i = 1, 2, \ldots, n\}$ where,

(5.1) $L_{X_i}^{\lambda}$ is the *strategy set* of player i, and

(5.2) $V_i^{\lambda} : \Omega \times \prod_{i=1}^{n} L_{X_i}^{\lambda} \to \mathbf{R}$ is the *payoff function* of player i.

Each $\overline{\mathbf{B}}^{\lambda}$ satisfies the assumptions of Scarf's α-core existence theorem and therefore there exists $x^{\lambda} \in \prod_{i=1}^{n} L_{X_i}^{\lambda}$ satisfying the following property:

For μ-almost all $\omega \in \Omega$, it is not true that there exist $S \subset \{1, 2, \ldots, n\}$ and $(y_i)_{i \in S} \in \prod_{i=1}^{n} L_{X_i}^{\lambda}$ such that for each $z^{I/S} \in \prod_{i \notin S} L_{X_i}^{\lambda}$, $V_i^{\lambda}(\omega, (y^S, z^{I/S})) > V_i^{\lambda}(\omega, x^{\lambda})$ for all $i \in S$.

Since the set Λ is directed by inclusion we have constructed a net $\{(x_1^{\lambda}, x_2^{\lambda}, \ldots, x_n^{\lambda}) : \lambda \in \Lambda\}$ of α-core strategies in $\prod_{i=1}^{n} L_{X_i}$. Since by Claim 5.1 each L_{X_i} is weakly compact so is $\prod_{i=1}^{n} L_{X_i}$. Hence the net $\{(x_1^{\lambda}, x_2^{\lambda}, \ldots, x_n^{\lambda}) : \lambda \in \Lambda\}$ has a subset which converges weakly to $(x_1, x_2, \ldots, x_n)$ in $\prod_{i=1}^{n} L_{X_i}$. We must show that $x_1, x_2, \ldots, x_n$ is an α-core strategy for **B**. Adopting a similar argument with that used in the proof of Theorem 3.1, one can now complete the proof of Theorem 3.2.

6. CONCLUDING REMARKS

Remark 6.1: In Theorems 3.1 and 3.2, Y is assumed to be a separable Banach lattice with order continuous norm whose dual Y^* has the RNP. Basic examples of spaces which satisfy the above properties are:

(i) the Euclidean space $\mathbf{R}^l$,

(ii) the space l^p $(1 < p < \infty)$ of real sequences $\{a_n : n = 1, 2, \ldots\}$ for which the norm $\|a_n\|_p = (\sum_{n=1}^{\infty} |a_n|^p)^{\frac{1}{p}}$ is finite,

(iii) the space $L^p(\Omega, \mathbf{F}, \mu)$ $(1 < p < \infty)$ of measurable functions f on the measure space $(\Omega, \mathbf{F}, \mu)$ for which the norm $\|f\|_p = (\int_{\omega \in \Omega} |f(\omega)|^p d\mu(\omega))^{\frac{1}{p}}$ is finite.

It is important to give examples of spaces that Theorems 3.1 and 3.2 do not cover:

(iv) $L_1[0, 1]$ or $L_1(\mu)$, if μ is not purely atomic, c_0, l_∞, $L_\infty[0, 1]$ and

(v) the space $C(X)$ of continuous real-valued functions on the infinite compact Hausdorff space X (with the supremum norm).

Recall that the spaces in (iv) and (v) do not have the RNP moreover, order intervals are not weakly compact in $L_\infty[0, 1]$ and $C(X)$.

Remark 6.2: The separability assumption on Y was used in order to make the Aumann measurable selection theorem applicable. The latter result was used in several steps in the proofs of Theorems 3.1 and 3.2. The relaxation of the separability of Y is possible. In this case however, the consumption set L_{X_i} will be the set of all *Gel'fand integrable selections from the set-valued function* $X_i : \Omega \to 2^{Y^*}$, and one will need to appeal to results on the existence of weak* measurable selections.

Remark 6.3: Theorem 3.1 and its Corollaries can be easily extended to coalition production economies provided that the production technology is assumed to be balanced. The proof remains essentially unchanged.

Remark 6.4: Kahn and Mookerjee (1989), have introduced a core-like concept in order to analyse games in normal form with differential information. Their concept in a two-person game, coincides with the coalitional Nash equilibrium. No existence results are given in their paper. However, it is known [see, for instance, Scarf (1971)] that even if preferences are strictly convex and continuous the set of coalitional Nash equilibrium strategies may be empty.

Remark 6.5: We conjecture that the core of a large finite private information economy will converge to the standard Debreu-Scarf (1963) core notion, with the approximation getting finer the larger the private information economy, (this will follow from the law of large numbers provided there is some kind of independence among agents). Hence, we can conclude that core allocations in large private information economy will become Walrasian. We also conjecture that without the independence assumption among agents, core allocations in a

large private information economy will characterize some kind of rational expectations equilibrium.[7]

[7] Kobayashi (1980, p. 1647) has made a related conjecture for the syndicate problem. Moreover, Srivastava (1984) has shown that a Wilson-type core allocation in a differential information economy becomes a full information core allocation as the number of agents in the economy tends to infinity.

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